KUTISH JOYLARI SONI CHEKLANGAN SISTEMALAR NOSTATSIONAR NAVBAT UZUNLIKLARI TAQSIMOTLARI UCHUN IKKILANMA MUNOSABAT

Axmatov N. B., Temirov E. I.
Samarqand temir yo'l texnikumi matematika fani o'qituvchisi

Quyidagicha xarakterlanadigan $G_1|G_2|1|N-F_1$ va $G_2|G_1|1|N-F_2$ bir kakalli xizmat ko'rsatish sistemalarini qaraymiz:

1. F_1 sistema$-t_n$ xizmat jarayoni boshlangandan keyin n –talabning sistemaga kelib tushish momenti va $z_n = t_n - t_{n-1}, n \geq 1, t_0 = 0$ bo'lsin. Faraz qilaylik, $z_1, z_2, \dots, z_n$lar bog'liq bo'lmagan va bir xil $A(t) = P(z_n < t), n \geq 1$, taqsimot funksiyasiga ega bo'lgan tasodifiy miqdorlar bo'lsin.

Talablarga ularning kelish tartibida xizmat ko'rsatiladi. Xizmat ko'rsatish vaqti uzunliklari $s_1, s_2, \dots, s_n, \dots$ lar bog'liq bo'lmagan bir xil $B(t) = P(s_n < t), n \geq 1$, taqsimot funksiyasiga ega bo'lgan tasodifiy miqdorlarni tashkil etsin.

Kutish joylari soni N ga teng ($N \geq 1$).

2. F_2 sistema $-F_1$ sistemada $A(t)$ va $B(t)$ taqsimot funksiyalarini o'zaro almashtirib, ikkinchi F_2 sistemani hosil qilamiz. Bu yerda agar t_n^* –xizmat jarayoni boshlangandan keyin n –talabning sistemaga kelib tushish momenti bo'lsa, $z_n^* = t_n^* - t_{n-1}^*, n \geq 1$, tasodifiy miqdorlar $B(t) = P(z_n^* < t), n \geq 1$, taqsimotga ega bo'ladi. Shuningdek, n –talabga xizmat ko'rsatish vaqti uzunligi s_n^* tasodifiy miqdorning taqsimoti $A(t) = P(s_n^* < t), n \geq 1$, bo'ladi.

Yuqorida ta'riflangan F_1 va F_2 sistemalar ikkilanma xizmat ko'rsatish sistemalari deyiladi.

Quyidagi belgilashlarni kiritaylik:

$\xi_1(t)$ va $\xi_2(t)$ –mos holda F_1 va F_2 sistemalarda t momentda mavjud bo'lgan talablar soni;

$$P_k^{(j)}(t) = P\left(\xi_1(t = k, \quad \sup_{0 \leq u \leq t} \xi_1(u) \leq N, \inf_{0 \leq u \leq t} \xi_1(u) > 0 / \xi_1(0) = j\right),$$

$k = \overline{1, N}$, $j = \overline{1, N}$.

$Q_k^{(j)}(t) = (P(\xi_2(t) = k, \sup_{0 \leq u \leq t} \xi_2(u) \leq N, \inf_{0 \leq u \leq t} \xi_2(u) > 0 / \xi_2(0) = j), \quad k = \overline{1, N}, j = \overline{1, N}.$

Teorema. $k = \overline{1, N}$ va $j = \overline{1, N}$ lar uchun ushbu munosabat o'rinli:

$$P_k^{(j)}(t) = Q_j^{(k)}(t) \qquad (1)$$

Ushbu munosabat $N = \infty$ uchun ham o'rinli.

Izoh. (1) munosabat

$A(t) = 1 - e^{-\lambda t}, \lambda > 0, t > 0$

hol uchun [4] ishda keltirilgan natijaning umumlashmasi hisoblanadi. Shuningdek, (1) munosabatga o'xshash tenglik [3] ishda M|G|1|N va GJ|M|1|N-1 sistemalar o'rtasida ham ko'rsatilgan.

Teoeremaning isboti. Avval F_1 sistemani qaraylik. $\eta_1(t)$ va $\eta_2(t)$ mos holda $(0, t)$ vaqt oralig'ida kelib tushgan va xizmati tugallangan talablar soni bo'lsin. U holda ushbu munosabatlar o'rinli:

$k \leq j$ da

$P_k^{(j)}(t) = \sum_{m=0}^{\infty} P\left(\eta_1(t) = m,\ \eta_2(t) = j-k+m,\ sup_{0\le u\le t}\xi_1(u) \le N,\ inf_{0\le u\le t}\xi_1(u) > 0\ \right)$, (2)

$k > j$ da

$P_k^{(j)}(t) = \sum_{m=0}^{\infty} P(\eta_1(t) = k-j+m,\ \eta_2(t) = m,\ sup_{0\le u\le t}\xi_1(u) \le N,\ inf_{0\le u\le t}\xi_1(u) > 0\ )$, (3)

Agar

$(\eta_1(t) = m,\ sup_{0\le u\le t}\xi_1(u) \le N) = (z_1 + \cdots + z_m < t < z_1 + \cdots + z_{m+1})$

va

$(\eta_2(t) = m,\ \underset{0\le u\le t}{inf}\ \xi_1(u) > 0) = (s_1 + \cdots + s_m < t < s_1 + \cdots + s_{m+1})$

tengliklarni e'tiborga olsak, $k \le j$ da quyidagi munosabatlar o'rinli bo'ladi:

$$P\left(\eta_1(t) = m,\quad \eta_2(t) = j-k+m,\ \underset{0\le u\le t}{sup}\ \xi_1(u) \le N,\ \underset{0\le u\le t}{inf}\ \xi_1(u) > 0\ \right) =$$

$$= P\left(z_1 + \cdots + z_m < t < z_1 + \cdots + z_{m+1}, s_1 + \cdots + s_{j-k+m} < t < s_1 + \cdots + s_{j-k+m+1}\right).$$

$s_1, s_2, \ldots$ tasodifiy miqdorlar kiruvchi talablar oqimiga bog'liq bo'lmaganligi sababli ushbu tenglikdan quyidagi munosabatga ega bo'lamiz:

$$P\left(\eta_1(t) = m,\quad \eta_2(t) = j-k+m,\ \underset{0\le u\le t}{sup}\ \xi_1(u) \le N,\ \underset{0\le u\le t}{inf}\ \xi_1(u) > 0\ \right) =$$

$$P\left(z_1 + \cdots + z_m < t < z_1 + \cdots + z_{m+1}\right) \cdot P\left(s_1 + \cdots + s_{j-k+m} < t < s_1 + \cdots + s_{j-k+m+1}\right) =$$

$$= [P(z_1 + \cdots + z_m < t) - P(z_1 + \cdots + z_{m+1} < x)] \cdot \left[P\left(s_1 + \cdots + s_{j-k+m} < t\right) - P\left(s_1 + \cdots + \right.\right.$$

$$\left.\left. + s_{j-k+m+1}\right) - P(s_1 + \cdots + s_{j-k+m+1} < t)\right] \qquad (4)$$

Agar $\{z_n\}_{n=1}^{\infty}$ va $\{s_n\}_{n=1}^{\infty}$ lar bog'liq bo'lmagan va mos holda $A(t)$ va $B(t)$ taqsimotlarga ega bo'lgan tasodifiy miqdorlar ketma-ketligini tashkil etishni e'tiborga olsak, (4) tenglik quyidagi ko'rinishga ega bo'ladi:

$$P(\eta_1(t) = m,\ \eta_2(t) = j-k+m,\ sup_{0\le u\le t}\xi_1(u) \le N,\ inf_{0\le u\le t}\xi_1(u) > 0\ ) = \left[A^{*(m)}(t) - \right.$$

$$\left. - A^{*(m+1)}(t)\right] \cdot \left[B^{*(j-k+m)}(t) - B^{*(j-k+m+1)}(t)\right], \qquad (5)$$

bu yerda $A^{*(m)}(t)$ va $B^{*(m)}(t)$-mos holda $A(t)$ va $M(t)$ funksiyalarning k karrali bog'lanmasi.

$$P(\eta_1(t) = k-j+m,\ \eta_2(t) = m,\ sup_{0\le u\le t}\xi_1(u) \le N,\ inf_{0\le u\le t}\xi_1(u) > 0\ ) = \left[A^{*(k-j+m)}(t) - \right.$$

$$\left. A^{*(k-j+m+1)}(t)\right] \cdot \left[B^{*(m)}(t) - B^{*(m+1)}(t)\right], \qquad (6)$$

(5) va (6) tengliklarga ko'ra (2) va (3) munosabatlar quyidagi ko'rinishga ega bo'ladi:

$$P_k^{(j)}(t) = \begin{cases} \sum_{m=0}^{\infty}\left[A^{*(m)}(t) - A^{*(m+1)}(t)\right] \cdot \left[B^{*(j-k+m)}(t) - B^{*(j-k+m+1)}(t)\right], & k \le j \text{ da} \\ \sum_{m=0}^{\infty}\left[A^{*(k-j+m)}(t) - A^{*(k-j+m+1)}(t)\right] \cdot \left[B^{*(m)}(t) - B^{*(m+1)}(t)\right], & k > j \text{ da} \end{cases} \qquad (7)$$

Yuqoridagi mulohazalarning takrorlab, $Q_k^{(j)}(t)$ funksiya uchun ushbu munosabatni hosil qilamiz:

$$Q_k^{(j)}(t) = \begin{cases} \sum_{m=0}^{\infty}\left[B^{*(m)}(t) - B^{*(m+1)}(t)\right] \cdot \left[A^{*(j-k+m)}(t) - A^{*(j-k+m+1)}(t)\right], & k \le j \text{ da} \\ \sum_{m=0}^{\infty}\left[B^{*(k-j+m)}(t) - B^{*(k-j+m+1)}(t)\right] \cdot \left[A^{*(m)}(t) - A^{*(m+1)}(t)\right], & k > j \text{ da} \end{cases} \qquad (8)$$

(7) va (8) munosabatlardan (1) tenglik kelib chiqadi.

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